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The problem of scattering by a triangular pyramid is studied using an extension of the geometrical theory of diffraction by the method of moments. Thus, the current distribution on the faces of the perfectly conducting pyramid is found by solving the three dimensional wedge diffraction problem once for each wedge, or a total of six times. This procedure yields the current in the GTD region on each face. The current near the edge of each wedge is then found using the moment method. The currents so obtained do not include that current caused by tip diffraction. The effect of this current on the far zone field may →

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be added post facto using a formula given by Keller, et al.

Results are shown for the bistatic radar cross section and in all cases the results are in good agreement with experimental measurements.

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I. INTRODUCTION

In recent years the geometrical theory of diffraction [1] has become one of the more useful methods for finding solutions to antenna and scattering problems. The geometrical theory of diffraction is, however, limited to geometries for which the diffraction coefficient is known.

Quite recently a technique has been developed for extending the GTD by the method of moments [2]. In this technique the diffraction coefficient is treated as an unknown, thereby permitting a larger class of problems to be treated with the GTD. In the case of planar geometries such as the wedge, this procedure has been improved by Sahalos and Thiele [3] using a three term representation of the diffraction coefficient. Using this improved technique, it has been shown that one can find the current near a discontinuity in a surface, such as near the edge of a wedge, for an arbitrary incidence angle.

Previously, the extended GTD technique has been applied to 2-dimensional problems. In this report we will use the method to treat a 3-dimensional geometry, the three sided pyramid. The reason for treating this particular geometry is to demonstrate the applicability of the extended GTD technique to 3-dimensional geometries. As such, it represents a first step in applying the GTD-moment method technique to 3-dimensional problems. Other geometries and modifications of the method to be described in this report will be considered subsequently.

For the problem of scattering by a pyramid as treated in this report, each face of the pyramid is composed of two regions, a GTD region and a moment method region near the edges. The current distribution in the GTD region is found by solving the 3-dimensional wedge diffraction problem once for each wedge, or a total of six times. Next, the current near the edge of each wedge is found using the moment method. However, the currents obtained on the faces of the pyramid do not include that current caused by tip diffraction. The effect of this current on the far zone scattered field may be added post facto using a formula by Keller, et al [4]. While the effect of tip diffraction is not large [5], its inclusion does bring the theoretical results into closer agreement with measurements.

In the work done here, only first order diffraction is considered and this is found to be adequate for determining the far zone field. On the other hand, if one was interested in the near zone field, it would be imperative that at least second order diffraction be considered.

Section III presents the extended GTD formulation for the pyramid while Section IV gives results for the bistatic radar cross section as well as the currents on the cone. Some concluding remarks are made in Section V.

II. FORMULATION

Consider the geometry of the three dimensional wedge diffraction problem (Figure 1). The field at any point in space will be an expression of the form

$$\vec{U}(r,\phi) = \vec{V}(r,\phi+\phi') + \vec{V}(r,\phi-\phi') \tag{1}$$

where $V(r,\phi+\phi')$ defines the field due to the incident wave and $V(r,\phi-\phi')$ the field due to the reflected wave.

If we want to have an expression for the geometrical optics field and the diffracted field separately, then by the help of the uniform theory of the diffraction [1] we can see that:

$$\vec{V}(r,\phi^{\pm}) = \vec{V}^{*}(r,\phi^{\pm}) + \vec{V}^{B}(r,\phi^{\pm}) \tag{2}$$

where in the general situation the function V^B will be

$$V_B(L,\phi^{\pm}) = I_{-\pi}(L,\phi^{\pm}) + I_{+\pi}(L,\phi^{\pm}) \quad . \tag{3}$$

where $\phi^{+} = \phi+\phi'$ and $\phi^{-} = \phi-\phi'$.
The parameter L is not the distance of the observation point but a function of this distance depending upon the nature of the incident wave.

Now $I_{\pm\pi}(L,\phi^{\pm})$ has been given by Kouyoumjian as follows:

$$I_{\pm\pi}(L,\phi^{\pm}) \cong \frac{e^{-j(\beta L+\pi/4)}}{jn\sqrt{2\pi}} \sqrt{a} \cot\left\{\frac{\pi\pm\phi^{\pm}}{2n}\right\} e^{j\beta a} \int_{(\beta La)^{1/2}}^{\infty} e^{-j\tau^{2}} d\tau \tag{4}$$

where $\beta=2\pi/\lambda$ and $a=1+\cos(\phi^{\pm}-2n\pi N)$.

N is an integer, positive, negative or zero, whichever best satisfies the equations

$$2n\pi N-\phi^{\pm} = -\pi \quad , \text{ for } I_{-\pi} \tag{5}$$

and

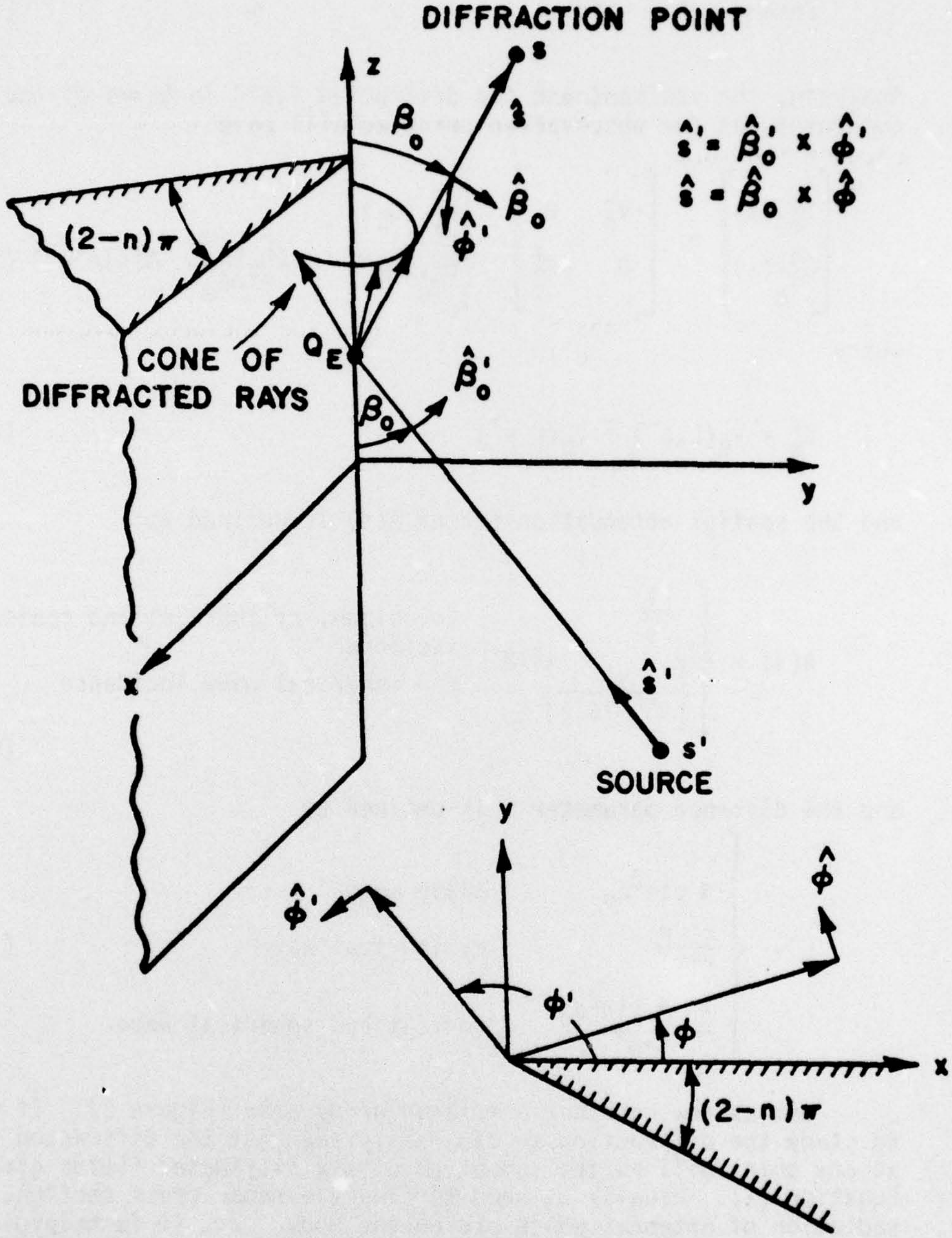

Figure 1. Geometry for three-dimensional wedge diffraction problem.

$$2n\pi N-\phi^{\pm} = +\pi, \quad \text{for } I_{+\pi} \quad . \tag{6}$$

Analyzing the incident and the diffracted field in terms of the polar components at the observation point we will have

$$\begin{bmatrix} E^d_{\phi'}(s) \\ E^d_{\beta_o'}(s) \end{bmatrix} = \begin{bmatrix} -V_B^- & 0 \\ 0 & -V_B^+ \end{bmatrix} \begin{bmatrix} E^i_{\phi'}(Q_E) \\ E_{\beta_o'}(Q_E) \end{bmatrix} \frac{\sqrt{L}\, e^{j\beta L}}{\sin\beta_o} A(s)e^{-j\beta s} \tag{7}$$

where

$$V_B^{\pm} = V_B(L,\phi^-) \mp V_B(L,\phi^+) \tag{8}$$

and the spatial attenuation factor A(s) is defined as

$$A(s) = \begin{cases} \dfrac{1}{\sqrt{s}} & \text{for plane, cylindrical and conical wave incidence} \\ \left\{\dfrac{s'}{s'(s+s')}\right\}^{1/2} & \text{for spherical wave incidence} \end{cases} \tag{9}$$

and the distance parameter L is defined as

$$L = \begin{cases} s\sin^2\beta_o & \text{plane wave} \\ \dfrac{r'\,r}{r+r'} & \text{cylindrical wave} \\ \dfrac{s'\,s\sin^2\beta_o}{s+s'} & \text{conical and spherical wave} \end{cases} \quad . \tag{10}$$

Let us now consider a triangular pyramid (Figure 2). If we want to study the diffraction we can easily see that the diffracted field at any point will be the summation of six diffracted fields given by Equation (7). Usually we need to find the radar cross section, or the radiation of antennas which are on the body. So, it is helpful to consider the current on the body. This current gives the necessary information for the electromagnetic properties of the body.

Because of the incident, reflected and diffracted fields, we suppose that on the body we have surface currents normal to the surface fields. The fields on the surface of the body can be computed by the help of the above equations. By knowing the surface current we can find

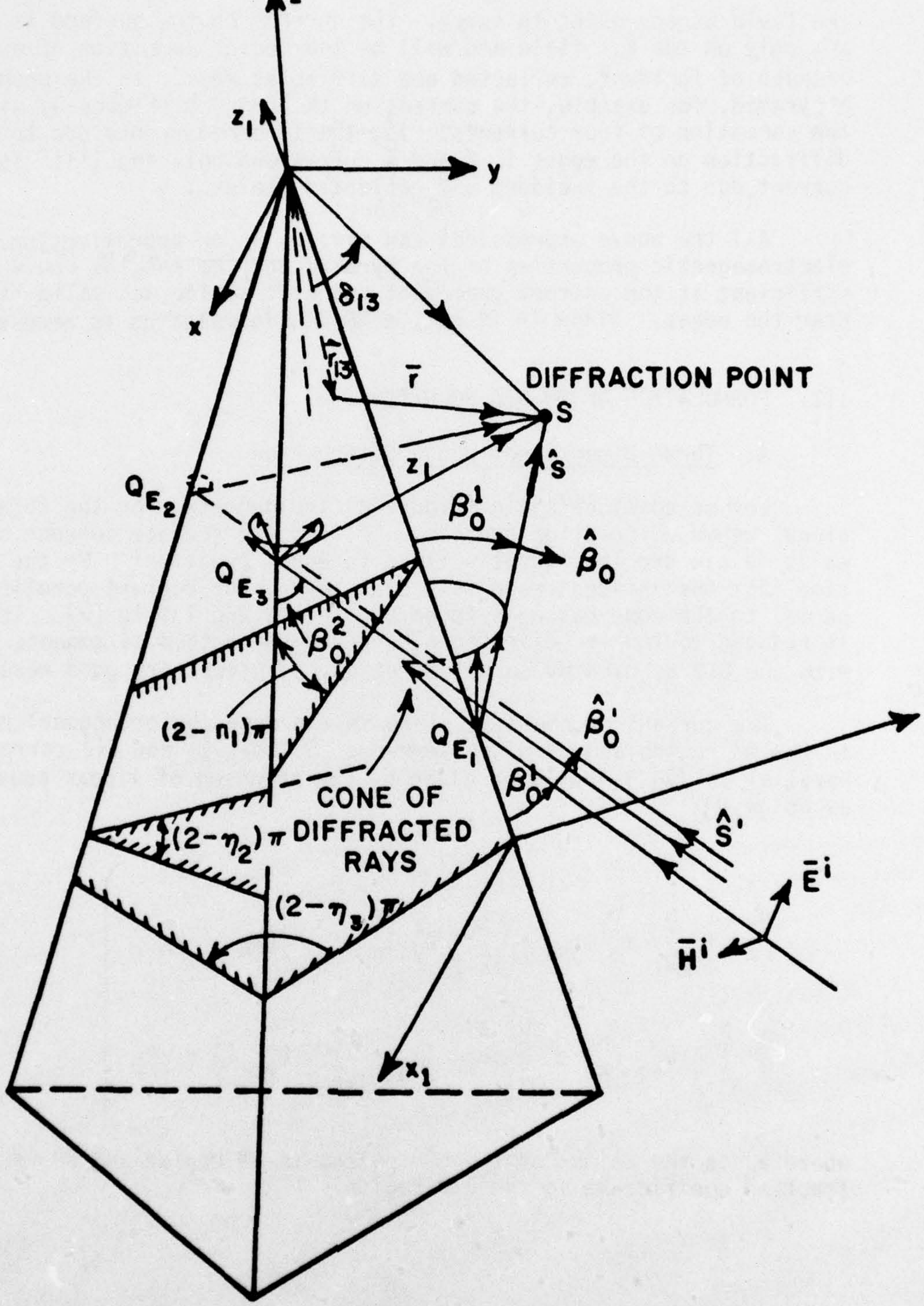

Figure 2. Geometry for the triangular pyramid.

the field at any point in space. The current on the surface is dependent only on the $\vec{E}_{\beta'}$ field and will be the vector summation of currents because of incident, reflected and diffracted rays. In the problem of a pyramid, for example, the current on the point M (Figure 3) will be the summation of four currents. I_d^1, I_d^2, I_d^4 are currents due to the diffraction on the edges 1, 2 and 4 correspondingly and I^i+I^r is the current due to the incident and reflected fields.

All the above expressions can give us is an approximation for the electromagnetic properties of the pyramid and the results could be sufficient if the uniform theory of the diffraction was valid for points near the edges. Since it is not, a MM-GTD formulation is necessary.

III. FORMULATION OF MM-GTD SOLUTION

A. Three Dimensional Wedge Diffraction

Let us consider again (Figure 4) the geometry for the three dimensional wedge diffraction problem. To find the surface current on the walls we can use the magnetic field integral equations. By the assumption that the incident wave is a plane wave, the current parallel and normal to the edge has been found by Sahalos and Thiele [2]. The problem is reduced to the two dimensional one and the method of moments along with the GTD as used by Burnside, et al [3] gives very good results.

The current on the X_1Y_1 plane is expressed by orthogonal pulses in the MM region and by GTD elsewhere. So the J_{X_1} and J_{Y_1} current parallel to (X_1,Y_1) will be given by the solution of linear equations as below[2]:

$$\left.\begin{aligned} &\frac{a_n^{X_1}}{2} + \sum_{m=1}^{N} a_m^{Y_1}\, I_{mn} + \sum_{\ell=-1}^{1} D_{y_1}^{\ell} I_n^{-\ell} = -H_z^i(x_n) - k_n \\ &\frac{1}{2}\sum_{n=-1}^{N} D_{x_1}^{n}\, E_{x_1}^{n} + \sum_{m=1}^{N} a_m^{Y_1}\, I_{m_D} + \sum_{\ell=-1}^{1} D_{y_1}^{\ell}\, I_D^{\ell} = -k_D \end{aligned}\right\} \tag{11}$$

where a_n is the weight of the n^{th} pulses in MM region and D^n is the diffraction coefficient in the GTD region.

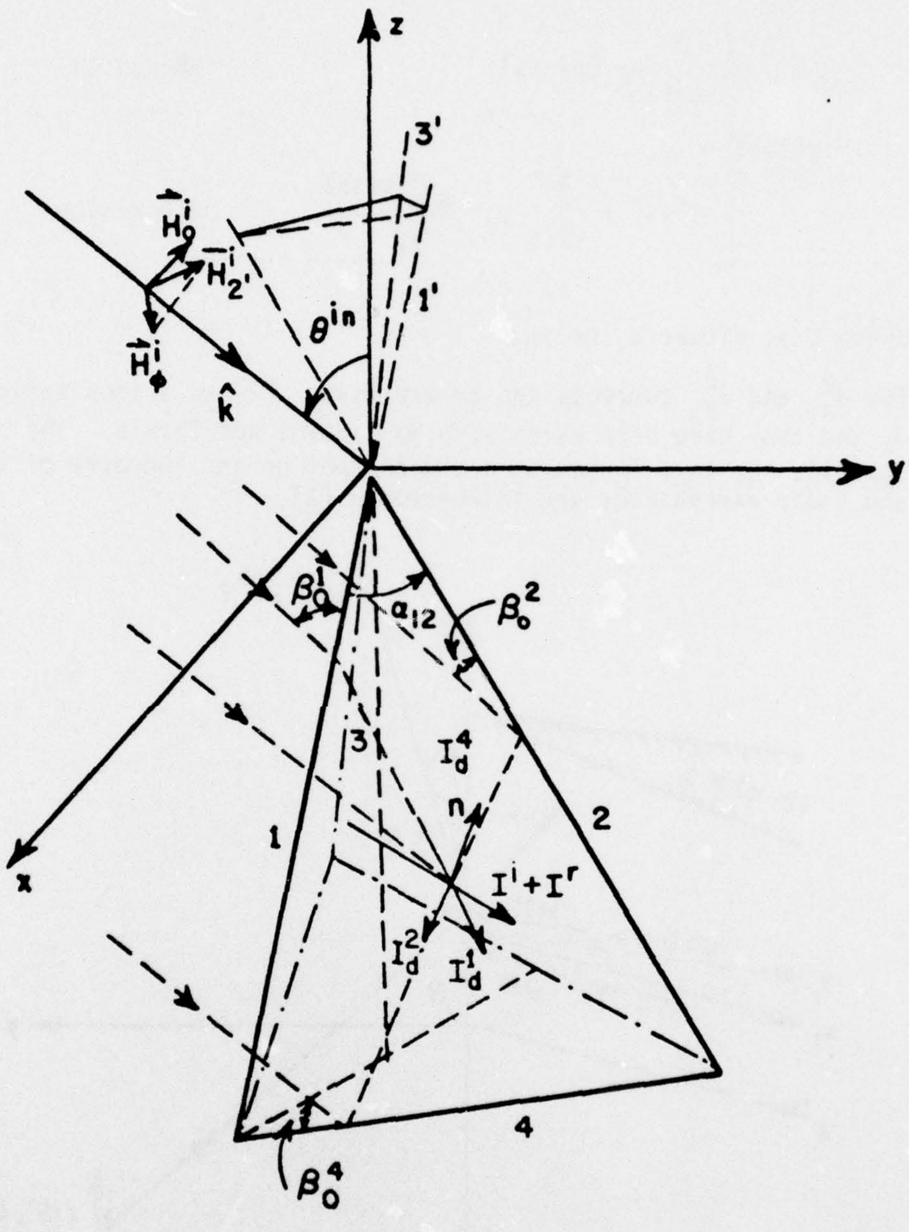

Figure 3. Geometry for one face of the triangular pyramid.

The current is:

$$J^{\ell}_{\ell} = \begin{cases} \sum_{n=1}^{N} a_n \, P(\ell-\ell_n) & \text{MM-region} \\ J^i + J^r + \sum_{i=-1}^{1} D^i_{\ell} \dfrac{e^{-jk\cos\beta\ell}}{\sqrt{\ell}} & \text{GTD-region} \end{cases} \qquad (12)$$

where ℓ is either X_1 or Y_1.

The $J^2_{x_1}$ and $J^2_{y_1}$ currents can be expressed [2] as a combination of a_n, D_n and they have been given also by Sahalos and Thiele. The parameters I_{mn}, I^{ℓ}_n, k_n, I_{mD}, I^{ℓ}_D and k_D are dependent on the geometry of the wedge and their expressions are in Reference [3].

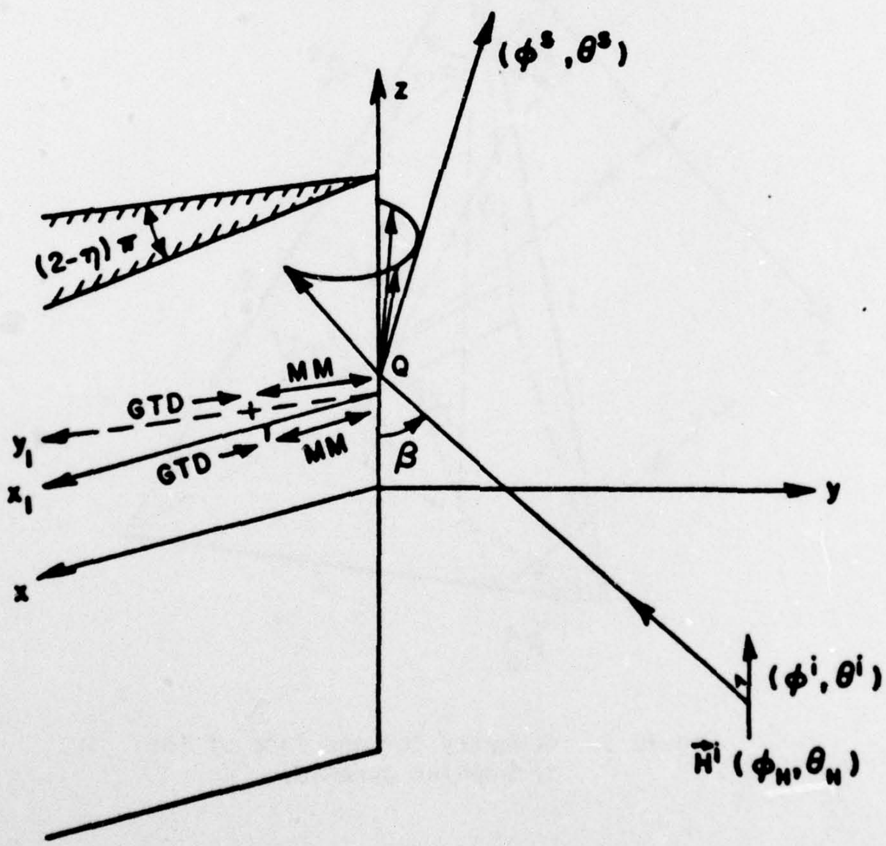

Figure 4. Geometry showing GTD and MM regions on both sides of a wedge.

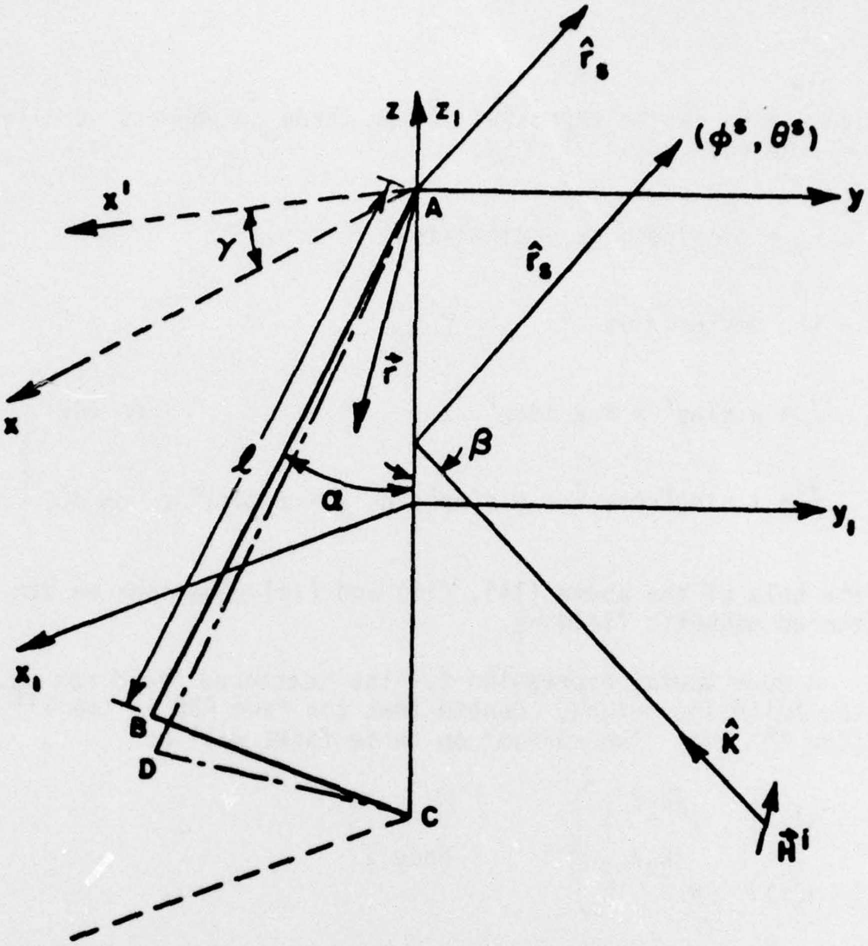

Figure 5. Geometry showing two faces (ABC and ACD) of the pyramid for far zone scattering.

Suppose that we want to find the far zone scattered field in the direction (ϕ^s, θ^s) due only to the surface ABCD which is composed of two triangular plates as shown in Figure 5. We will use the field expression

$$\vec{H}^s = -\frac{e^{jkr_o}}{r_o}\frac{k}{4\pi}\int_{(ABCD)}(\hat{r}_s \times \vec{J}_{\vec{r}})e^{jk\vec{r}\cdot\hat{r}_s}\, dS \tag{13}$$

where r_o is the distance between the origin point A and the observation point, $\vec{J}_{\vec{r}}$ is the current on the wall and k is the wavelength number. The current $\vec{J}_{\vec{r}}$ on the two faces ABC and ADC can be expressed as:

$$\left.\begin{aligned} \vec{J}_{\vec{r}} &= [J_z(x)\hat{z} + J_x(x)\hat{x}]\, e^{-jkz\cos\beta} \quad \text{on ABC} \\ \vec{J}_{\vec{r}} &= [J_z(x')\hat{z} + J_{y_1}(x)\cos\gamma\, \hat{x} - J_{y_1}(x')\sin\gamma\, \hat{y}]\, e^{-jkz\cos\beta} \quad \text{on ADC} \end{aligned}\right\} \tag{14}$$

The vector $\hat{r}_s$ can be expressed in the three components parallel to $\hat{x}$, $\hat{y}, \hat{z}$ by the help of ϕ^s, θ^s. So,

$$\hat{r}_s = \sin\theta^s\cos\phi^s \; \hat{x} + \sin\theta^s\sin\phi^s \; \hat{y} + \cos\theta^s \; \hat{z} \tag{15}$$

while the vector $\vec{r}$ is

$$\left.\begin{array}{ll} \vec{r} = x \sin\theta^r \; \hat{x} + z \cos\theta^r \; \hat{z} & \text{on ABC} \\ \vec{r} = x \sin\theta^r\cos\gamma \; \hat{x} - y \sin\theta^r\sin\gamma \; \hat{y} + z \cos\theta^r \; \hat{z} & \text{on ADC} \end{array}\right\} \tag{16}$$

By the help of the above (14), (15) and (16) equations we can find the scattered magnetic field $\bar{H}^s_f$.

A more useful expression for the scattered field can be obtained by the following method. Denote that the face ABC is the 1st face and ADC the 2nd one. The current on those faces will be:

$$\left.\begin{array}{l} J^1_z(x) \cdot e^{jk_z z} \; \hat{z} \\ J^1_x(x) \cdot e^{jk_z z} \; \hat{x} \end{array}\right\} \qquad \text{Face 1}$$

$$\left.\begin{array}{l} J^2_z(x') \; e^{jk_z z} \; \hat{z} \\ J^2_{x'}(x') \; e^{jk_z z} \; \hat{x}' \end{array}\right\} \qquad \text{Face 2}$$

We define the integral

$$A^i_w(\phi_s,\theta_s) = \frac{1}{jk(\cos\beta-\cos\theta_s)} \int_0^{\ell\sin a} J^i_w(x) \left\{ 2\, e^{jkx[(\cos\beta-\cos\theta_s)]\cot a-\sin\theta_s} - \left(1 + e^{jk\ell(\cos\beta-\cos\theta_s)}\right) \cdot e^{jkx\sin\theta_s\cos\phi_s} \right\} dx \; . \tag{17}$$

A vector $\vec{A}$ with the components

$$A_x(\phi_s,\theta_s) = A^1_x(\phi_s,\theta_s) + \cos\gamma \; A^2_x(\phi_s \pm \gamma, \theta_s) \qquad . \tag{18}$$

$$A_y(\phi_s,\theta_s) = -\sin\gamma\, A_x^2(\phi_s \pm \gamma, \theta_s) \tag{19}$$

$$A_z(\phi_s,\theta_s) = A_z^1(\phi_s,\theta_s) + A_z^2(\phi_s \pm \gamma, \theta_s) \tag{20}$$

can give the expression of $\vec{H}^s$.

The vector $\vec{A}(A_x,A_y,A_z)$ expressed in another cartesian system A $(x_o y_o z_o)$ will be given by using the Euler's angles. So

$$\begin{pmatrix} A_{x_o} \\ A_{y_o} \\ A_{z_o} \end{pmatrix} = e^{j\hat{k}\cdot\vec{R}_o}\,[K] \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \tag{21}$$

where R_o is the distance AA_o and K is the matrix of the directional cosines of the axis x,y,z to the axis x_o, y_o, z_o. The $\vec{H}^S$ field will be:

$$\begin{pmatrix} H^S_{x_o} \\ H^S_{y_o} \\ H^S_{z_o} \end{pmatrix} = -\frac{e^{j(\hat{k}\cdot\vec{R}_o - kr_o)}}{2r_o\lambda} \begin{bmatrix} 0 & -\cos\theta^S & \sin\theta^S\sin\phi^S \\ -\cos\theta^S & 0 & -\sin\theta^S\cos\phi^S \\ -\sin\theta^S\sin\phi^S & \sin\theta^S\cos\phi^S & 0 \end{bmatrix} \begin{pmatrix} A_{x_o} \\ A_{y_o} \\ A_{z_o} \end{pmatrix} \tag{22}$$

At this point we have only the scattered field from two triangular plates by using the diffraction from the common edge. In the next section we will use the above expressions to obtain the diffracted field from a pyramid.

B. Diffraction By a Pyramid

A pyramid is a combination of six edges and since the length of the edges are more than the wavelength the problem of the diffraction can be separated into six problems of wedge diffraction. For the six edges (see Figure 6) the scattered field will be the summation of six expressions like Equation (22).

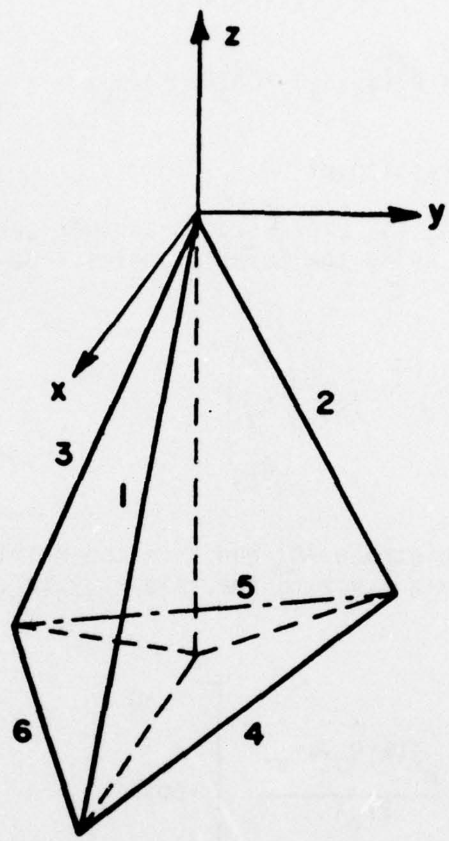

Figure 6. Six diffracting edges of the triangular pyramid.

First, it is important to define the shadow region of the pyramid. With the help of Figure 7 we can see for example that

if $\phi^1_{12} > \pi$ the face 12 is shadowed

if $\phi^1_{12} < \pi - A_1$ the face 13 is shadowed .

A more general definition of the shadow region can be obtained in the following manner. We take the horizontal plane (456) and starting from the direction of the incident wave we measure the angles ϕ_1, ϕ_2 from the edges 1 and 2. We define the maximum of the ratios:

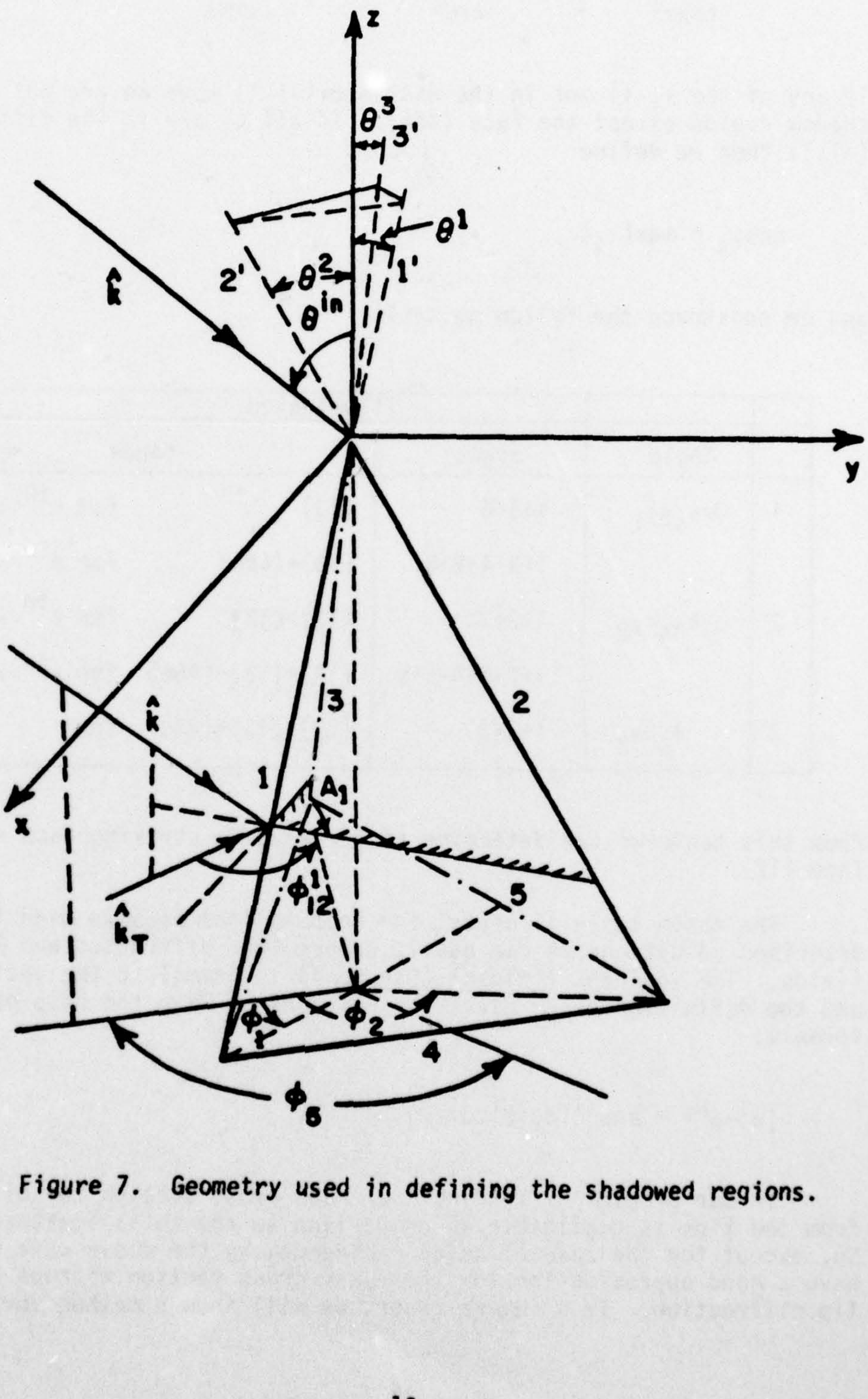

Figure 7. Geometry used in defining the shadowed regions.

$$\tau_1 = \frac{\tan\theta^{in}}{\tan\theta^1} \quad , \quad \tau_2 = \frac{\tan\theta^{in}}{\tan\theta^2} \quad , \quad \tau_3 = \frac{\tan\theta^{in}}{\tan\theta^3} \quad .$$

If any of the τ_i is not in the distance (-1,1) then we are not in the shadow region except the face (456). If all τ_i are in the distance (-1,1) then we define

$$\cos\phi_s = \max(\tau_i)$$

and we construct the following table:

		Illuminated		
	Angle	edges	faces	
1	$0<\phi_s\leq\phi_1$	1+3+6	(13)	for $\theta^{in}<\pi/2$
		1+3+4+5+6	(13)+(456)	for $\theta^{in}>\pi/2$
2	$\phi_1<\phi_s\leq\phi_2$	1+2+3	(12)+(13)	for $\theta^{in}<\pi/2$
		1+2+3+4+5+6	(12)+(13)+(456)	for $\theta^{in}>\pi/2$
3	$\phi_s>\phi_2$	1+2+3	(12)+(13)+(23)	for

From this table we can determine if the wave is striking face (23) or face (12).

The above table is useful for computations because with the described conditions we can easily program the diffracted and G.O. fields. The incident field $\bar{H}^i$ (Figure 4) is normal to the vector $\hat{k}$ and the definition of θ^H gives the value of ϕ^H with the help of the formula:

$$|\phi^i-\phi^H| = \cos^{-1}(\cot\theta^i\cot\theta^H) \quad . \tag{23}$$

In our problem of finding the radar cross section the diffraction from the tips is negligible in comparison to the total scattered field. So, except for the case of axial incidence, by the above work we can have a good approximation for the radar cross section without considering tip diffraction. In a future report we will show a method for finding

the tip diffraction. This will be done by using the GTD-MM formulation and the magnetic integral equation.

For bistatic scattering a modification to the method in this report for the scattered field could be obtained by adding the tip diffracted field as it is given by Keller, et al [4]. This formula gives the diffraction coefficient of a plane corner with two straight edges meeting at an angle and is of the form

$$C = \frac{j}{4\pi k_0} \frac{(\cos\theta+\cos\theta')\sin\delta}{(\cos a-\cos a')(\cos b-\cos b')} \qquad (24)$$

where a and b are the angles between the incident field and the two edges at the corner, a', b' are the angles between the diffracted field and the two edges; θ and θ' are the angles between the normal to the plane of the corner and the incident and diffracted rays; and δ is the angle between the two edges. Formula (24) along with expression (22) will give at least the far zone scattered field. It will be seen in the next section that inclusion of the tip diffracted field does noticeably raise the level of the radar cross section results at most aspect angles but the "scattering pattern" is little changed by the tip diffracted field.

IV. EXAMPLES

In all examples which follow we used a pyramid with the edges 1, 2,3 of length 9.144λ making an angle with the z-axis of 15°. At first the surface current was computed. In this current there is no diffracted current from the tips. The scattered field was computed two times, one without the tip diffraction and one with. By using Keller's formula the results as we will see are in closer agreement with the measurements than without the tip diffraction.

In Figures 8 and 9 we show the current on the faces of the pyramid (except the base) for an incident plane wave normal to edge 1 and with the magnetic field parallel to this edge. The current which is plotted shows only the diffracted current since the current of the geometrical optics field has been deleted from the curve.

In Figures 10, 11 and 12 we plotted the magnitude of the surface current of the pyramid for an incident plane wave normal to edge 4 and with the magnetic field parallel to this edge. The bistatic normalized electric field on the xy plane can be seen in Figure 13 for this case.

In Figures 14, 15, 16 and 17 we show the bistatic radar cross section. In all cases we keep the bistatic angle constant and rotate the pyramid about one axis. Figure 14 shows the bistatic cross section for the case of rotation about the z axis, with the transmitted signal

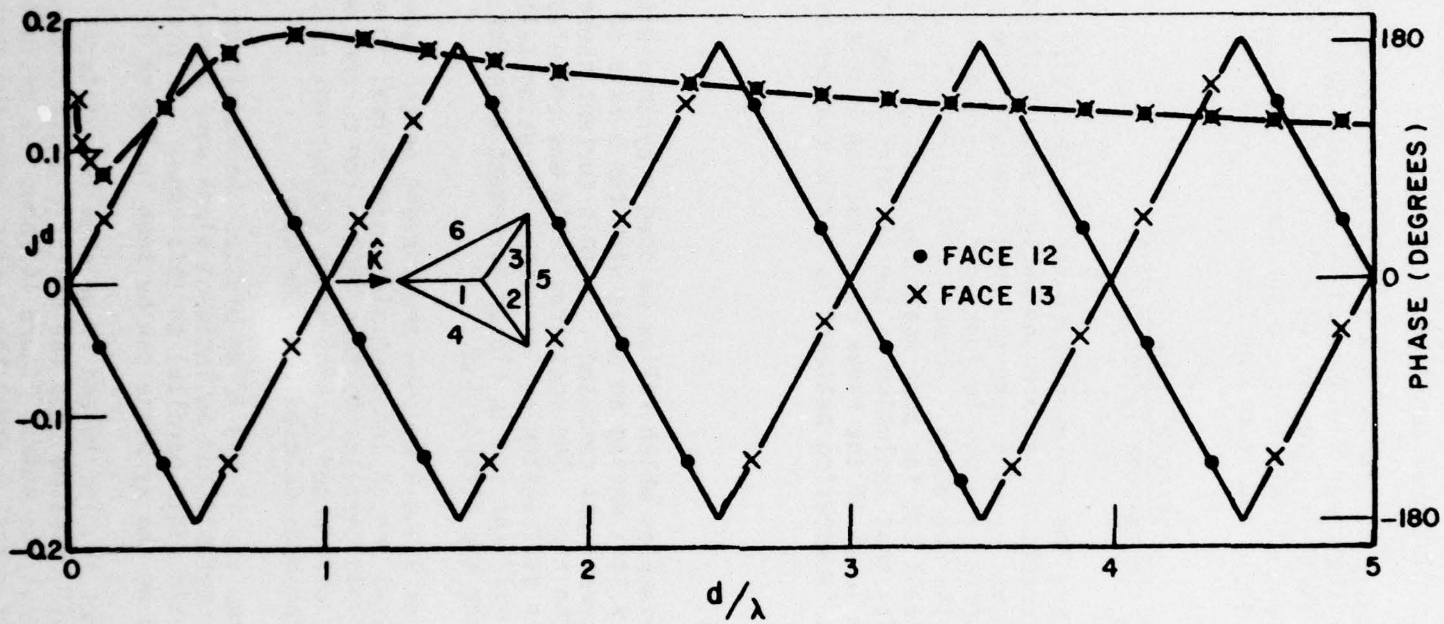

Figure 8. Current on two faces of the pyramid due to diffraction only.

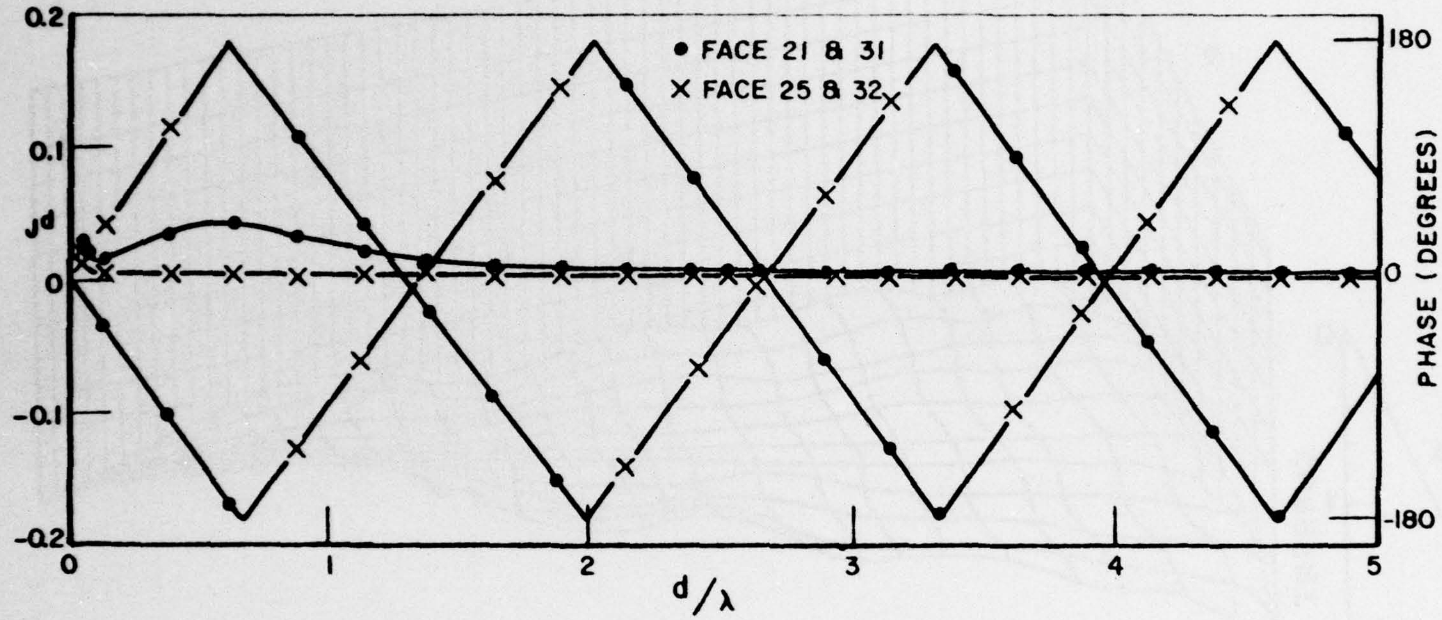

Figure 9. Current on two faces of the pyramid due to diffraction only.

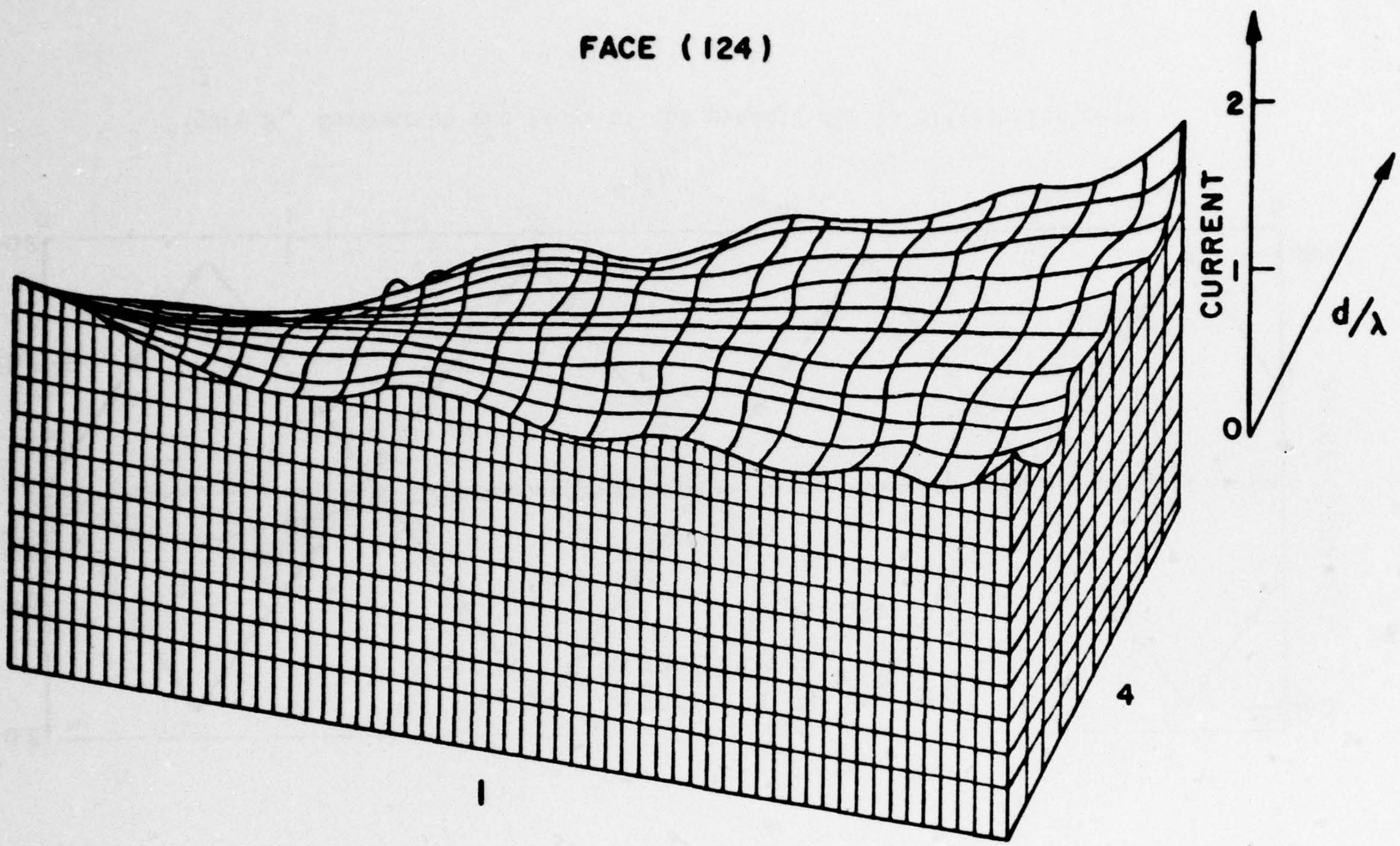

Figure 10. Magnitude of the surface current on face 124 of the pyramid.

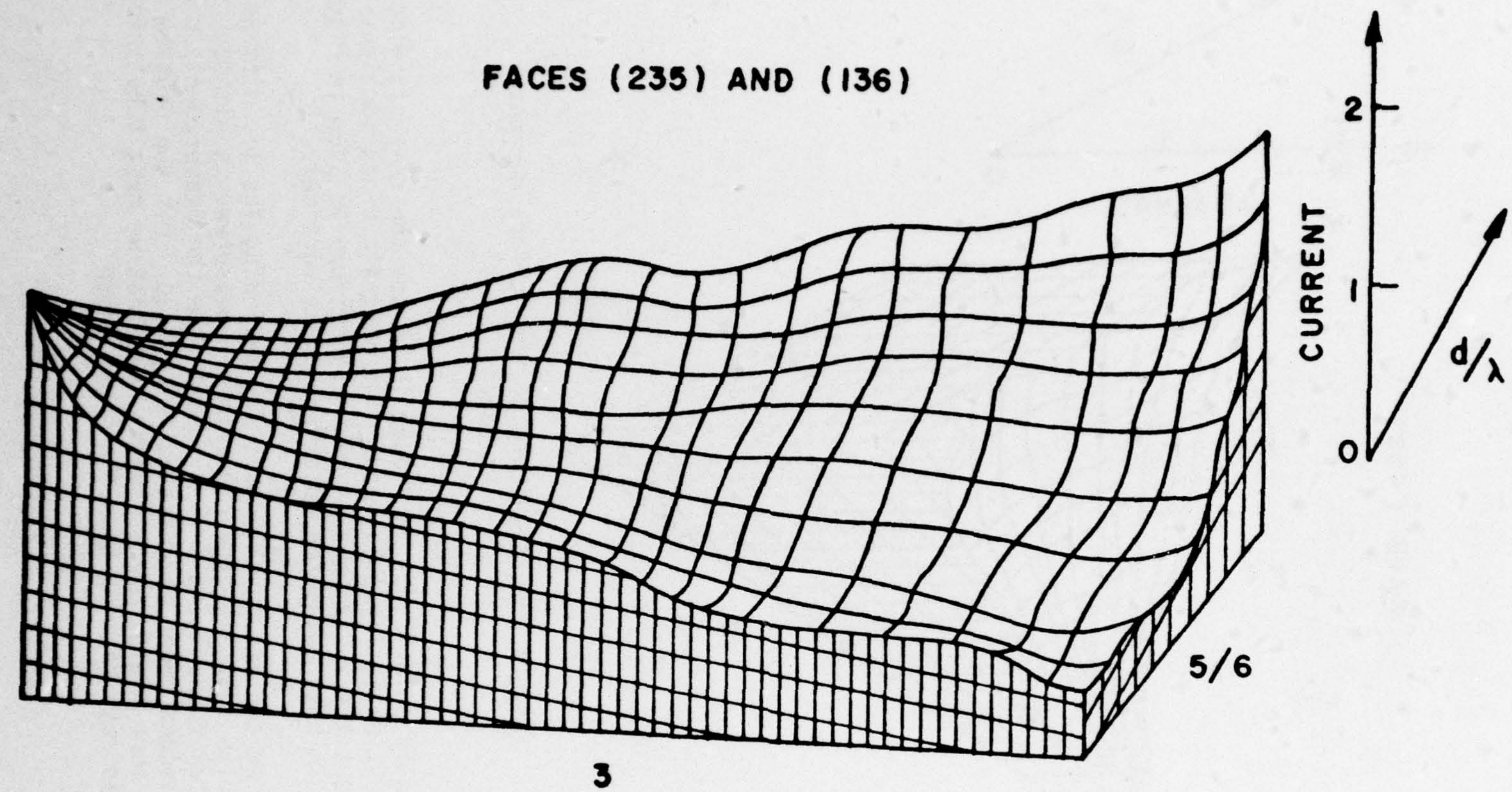

Figure 11. Magnitude of the surface current on either face 235 or 136.

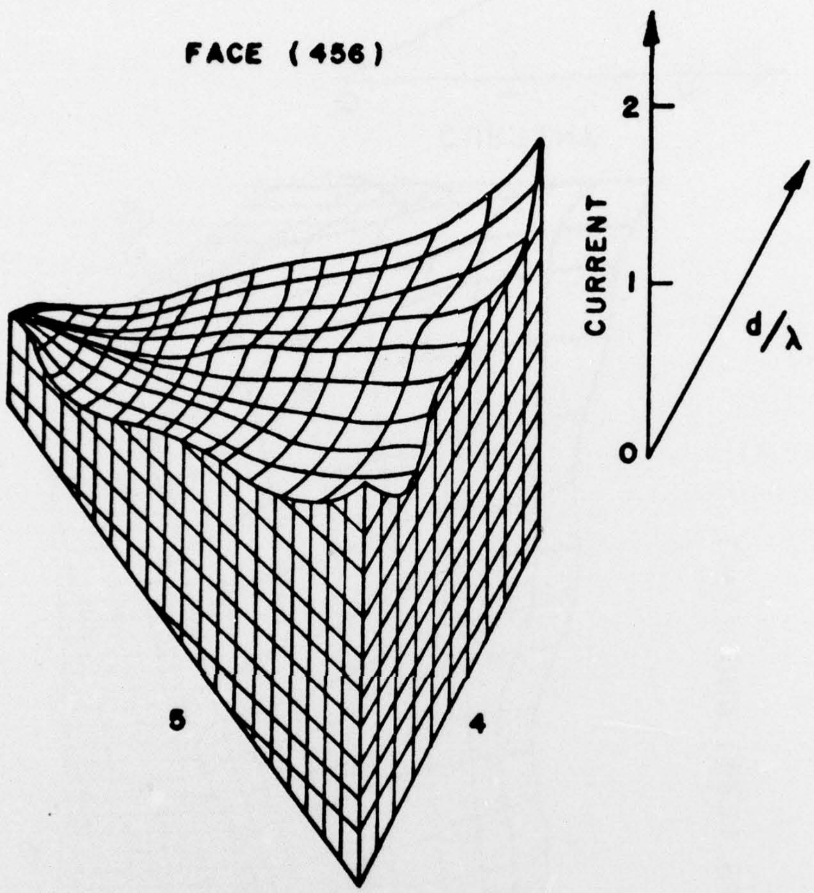

Figure 12. Magnitude of the surface current on face 456.

horizontally polarized. In Figure 15 we see the same situation but with the signal vertically polarized. The bistatic cross sections for rotations about the y- and x-axes can respectively be seen in Figures 16 and 17. In both cases the polarization is vertical.

As we can see in all cases, by including the tip diffraction we have results that are more close to the measurements although the theoretical results are a little down from the measurements probably because second order diffraction is ignored. For the problem of finding the near zone field, our analysis shows that we must take into account diffracted rays of more than the first order.

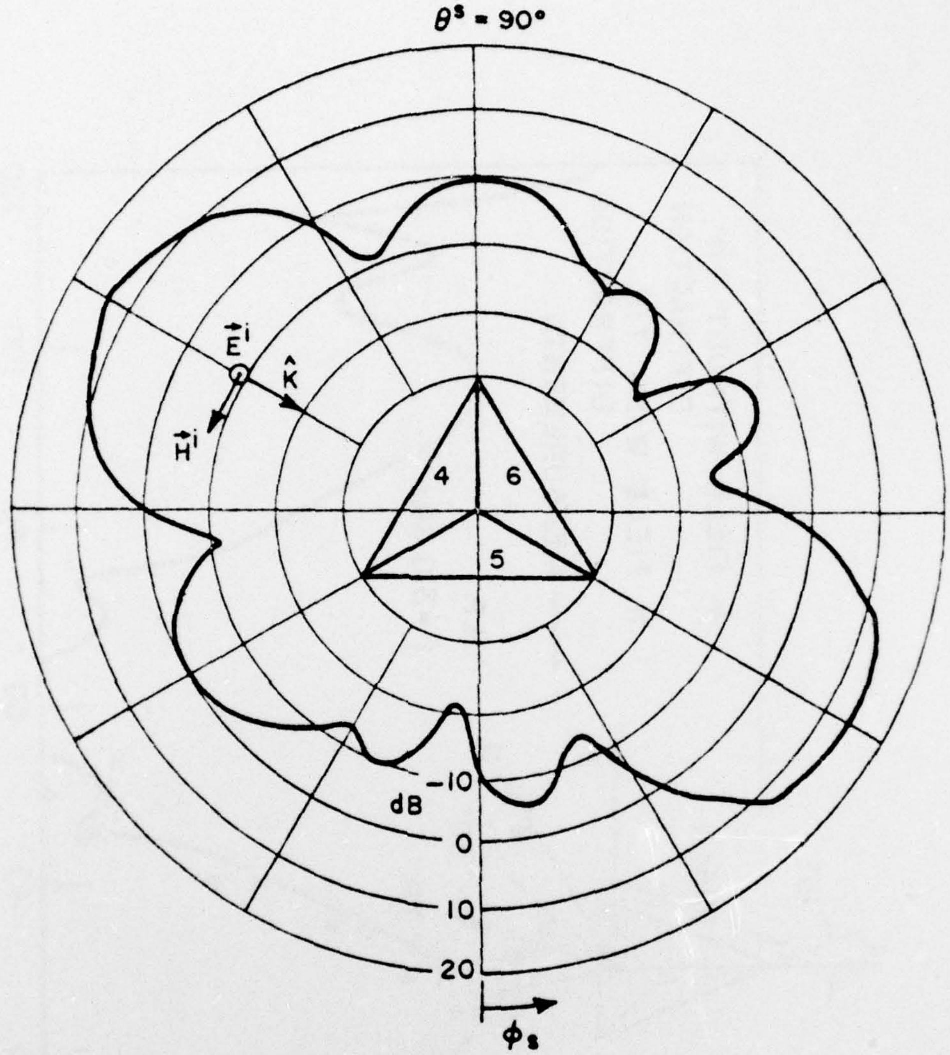

Figure 13. Scattered field pattern.

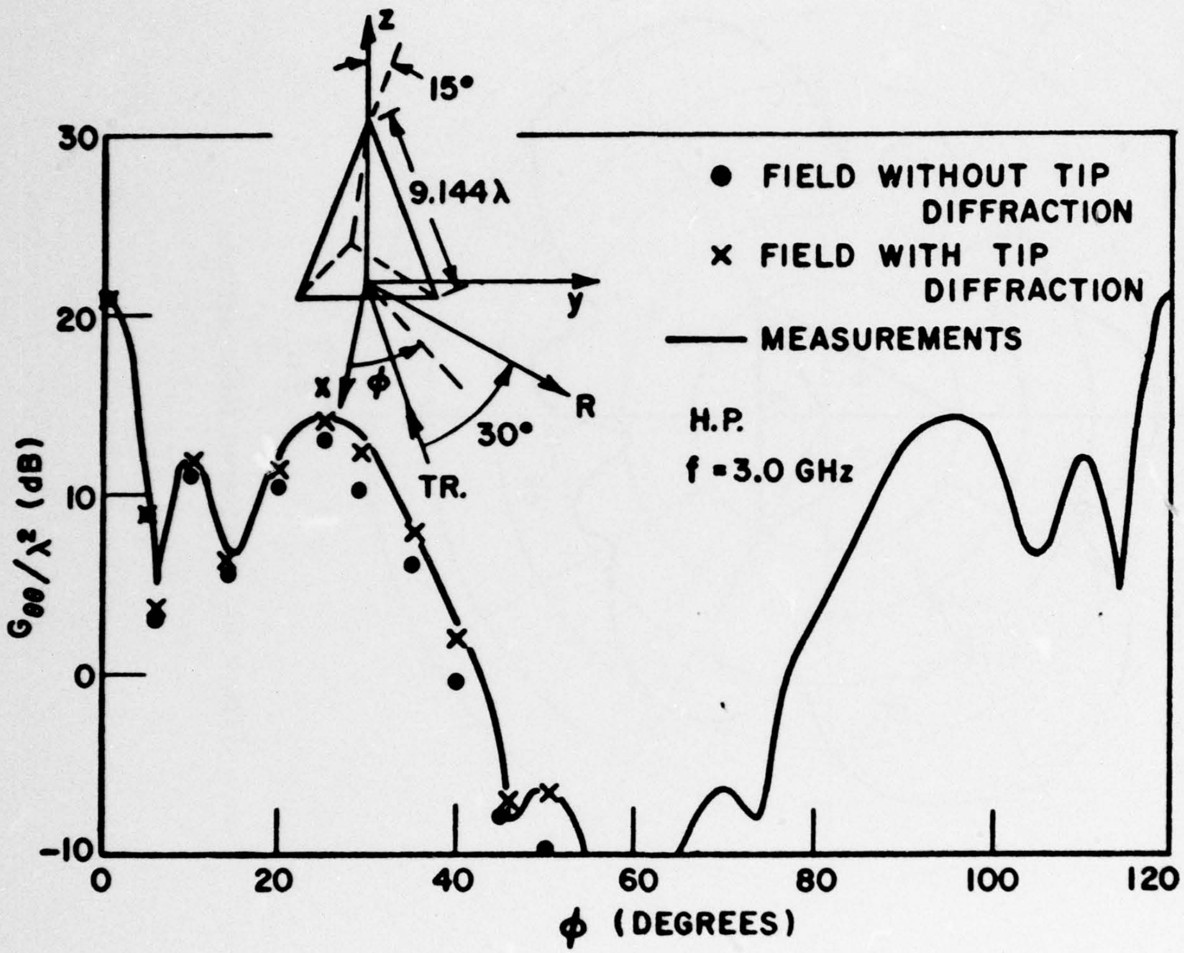

Figure 14. Bistatic (30°) radar cross section for ϕ polarization.

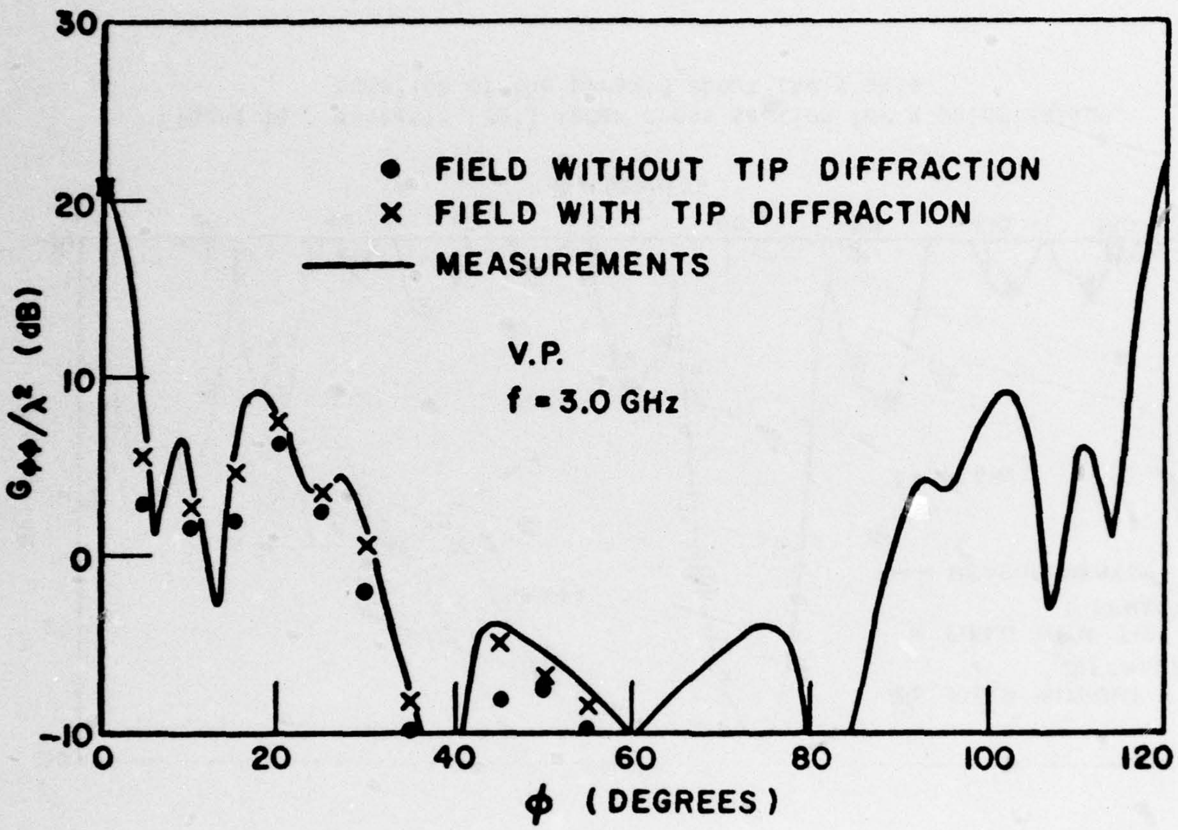

Figure 15. Bistatic (30°) radar cross section for z polarization.

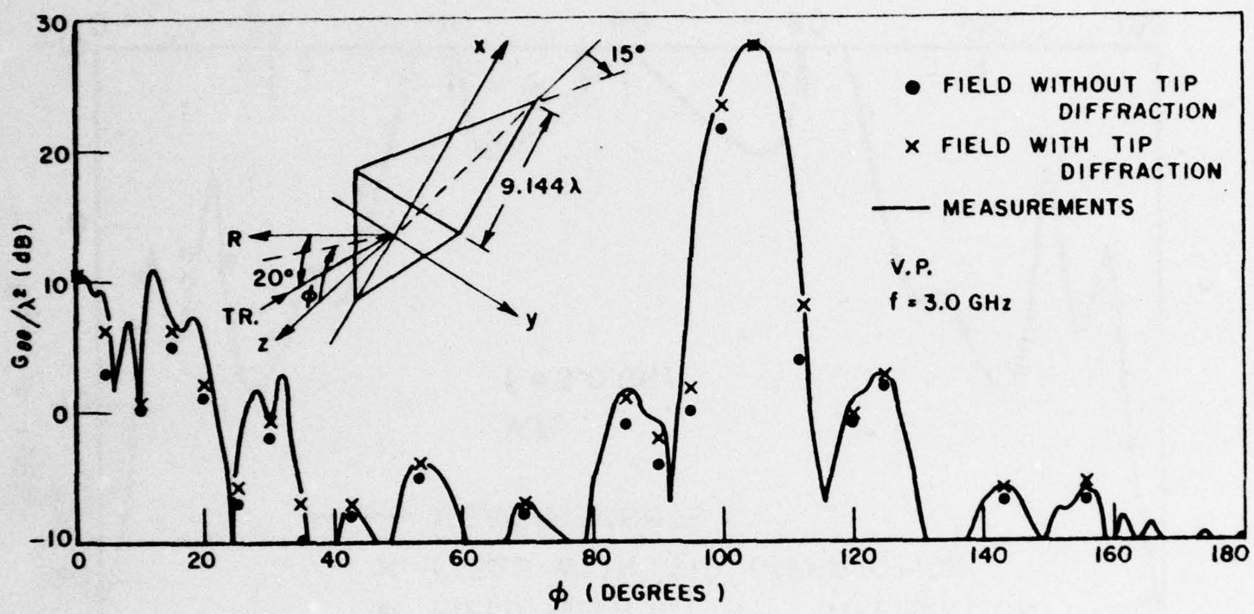

Figure 16. Bistatic (20°) radar cross section for x polarization, rotation of the pyramid about the y axis.

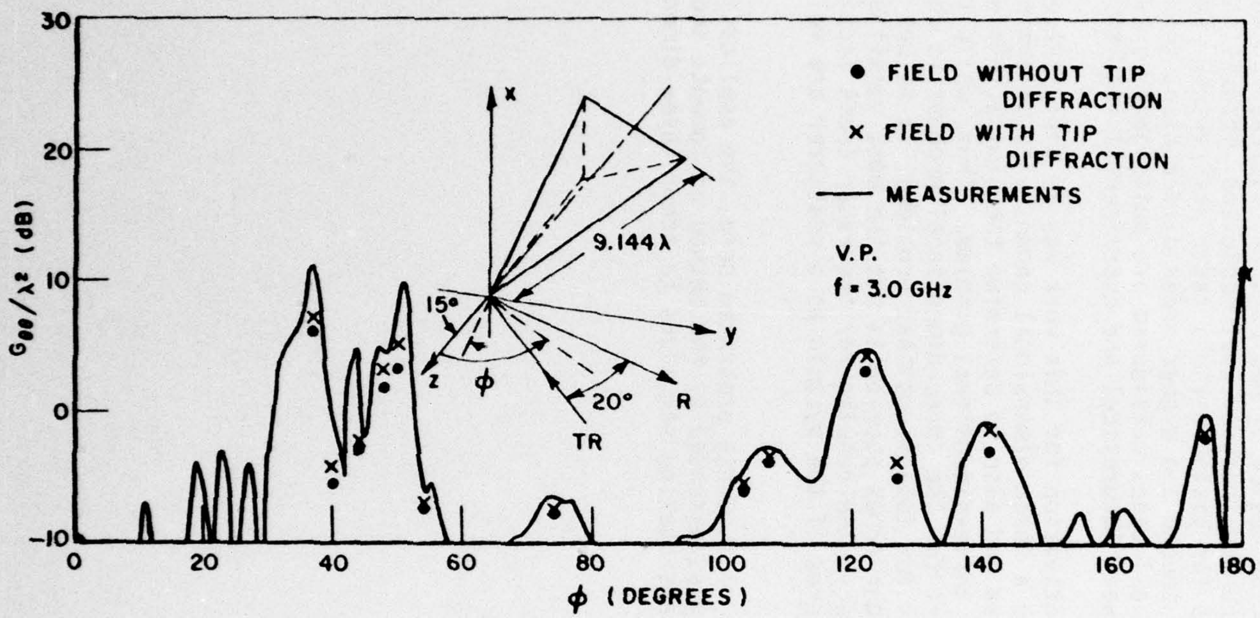

Figure 17. Bistatic (20°) radar cross section for x polarization, rotation of the pyramid about the x axis.

V. SUMMARY AND CONCLUSIONS

The work reported in this report has used the method of moments to extend the geometrical theory of diffraction such that a good approximate solution to the problem of scattering by a pyramidal cone is obtained. The solution basically neglects tip diffraction but it is shown that this effect can be included post facto and improves the results at most aspect angles for bistatic (15°) radar scattering. Even though the solution neglects second and higher order diffraction, the radar scattering results in all cases considered are quite good as evidenced by the agreement between theoretical and experimental results.

A primary motivation for this work was to apply the moment method - GTD technique to a three-dimensional geometry to determine the difficulties in doing so and also to determine the limitations of the method when applied to a three-dimensional problem. Thus, while the formulation in this report for the three-dimensional problem is obviously more involved than that for two-dimensional problems, it is not unduly so. And, while the radar cross section is not the most sensitive indicator of solution accuracy, the results for the radar cross section do indicate that all the features of the pyramid as a scatterer are well-predicted by the theory.

As a result of the work presented here, the application of the geometrical theory as extended by the method of moments should find use by other researchers seeking solutions to other three-dimensional problems.

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